GCE

Further Mathematics B (MEI)

Y432/01: Statistics minor

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f.
- Accept any value within a probability model that agrees with the correct value to 4 d.p.

Follow through should be used so that only one mark is lost for each distinct accuracy error.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1	(a)		Use of B(4, 0.2) soi P(X = 3) = 0.0256	M1 A1 [2]	3.3 1.1	$\frac{16}{625}$	2.56%
	(b)		E(X) = 0.8 oe Var(X) = 0.64 oe	B1 B1 [2]	1.1 1.1		
	(c)	(i)	Expected total = –100 + 0.8 × 50 = –60 p oe	M1 A1 [2]	3.1b 1.1	**M1** for 0.5(their $E(X)$) – 1 or 50(their $E(X)$) – 100 e.g. "loss of £0.60" With appropriate monetary unit	**SCB1** if following **M0** 60p seen N.B. -£0.60p is A0
	(c)	(ii)	Standard deviation = $50\times\sqrt{0.64} = 40$ p oe	B1 [1]	1.1	With appropriate monetary unit Mark final answer	N.B. £0.40p is A0

Question			Answer	Marks	AOs	Guidance	
2	(a)		$(1 - 0.12)^4 \times 0.12$ soi $= 0.0720$ awrt isw	**M1** **A1** **[2]**	**3.3** **1.1**	(0.0719634…)	NB 0.072 only is **A0**
	(b)		$0.88^9 = 0.3165$ awrt	**B1** **[1]**	**1.1**	(0.3164783…)	
	(c)		2 in first 19 then also 20th use B(19, 0.12) soi (to get 0.28026...) (0.28026... × 0.12 =) 0.0336 awrt	**B1** **M1** **A1** **[3]**	**3.1a** **2.2a** **1.1**	For identifying required approach Use of **correct** binomial distribution	(0.0336314…)
	(d)		$1 - 0.88^n \geq 0.95$ oe $n \geq \frac{\log 0.05}{\log 0.88} (= 23.43)$ oe soi (At least) 24 people	**M1*** **M1dep*** **A1** **[3]**	**3.1b** **1.1** **3.2a**	$0.88^n < 0.05$ $\log_{0.88} 0.05$ T & I: **both** 0.88^{23}=*0.0529* and 0.88^{24}=*0.0465* must be seen	For both M marks allow an equation or strict inequality **Must see method**

Question			Answer	Marks	AOs	Guidance	
3	(a)		Because a census would involve **destroying all** of the components.	**E1** **[1]**	**2.3**	"Testing all items is expensive" alone is **E0**	
	(b)		Because as more components are produced a change might develop in the production process. It is not a random method. It will not be representative of all the components.	**E1** **[1]**	**2.2b**	Only one reason needs to be given	**NB** "The sample is too small" is E0
	(c)		The sample should be: **unbiased** **representative** of the population chosen so that **components** are selected **independently**	 **B1** **B1** **[2]**	 **1.2** **1.2**	Any two reasons (**B1** each)	**NB** "The sample needs to be large" is B0

Question			Answer	Marks	AOs	Guidance	
4	(a)		Variance = 1.4212^2 = 2.0(198094…) Because the **variance** is close to the **mean** oe	**B1** **E1** **[2]**	**1.1** **2.2b**	1.98 ≈ 2.0 scores 2 marks **NB E1** only awarded if a relevant numerical comparison is seen	N.B. comparing 1.98 with 1.4212 is **B0 E0** For **E1** accept "Both are close to 2"
	(b)		For C3 probability = 0.273377 so C3 = 27.3377 For C7 probability = 0.050867 so C7 = 5.0867 D3 = $\frac{(28-their27.3377)^2}{their27.3377}$ = 0.0160	**M1** **A1** **M1** **A1** **[4]**	**3.4** **2.2a** **1.1a** **1.1**	For either C3 or C7 Allow second value to be found by subtraction Allow $\frac{(27.3377-28)^2}{27.3377}$	Values should all be to 4dp to score full marks Deduct at most one **A** mark if any values are not given to 4dp
	(c)		Because otherwise some of the **expected frequencies** would be too low (<5) (and so the test would not be valid)	**E1** **[1]**	**2.4**		
	(d)		H_0: **Poisson** model is a good fit H_1: **Poisson** model is not a good fit (Test statistic =) 1.37 Refer to χ^2_4 Critical value at 5% level = 9.488 1.37 < 9.488 Insufficient evidence to reject H_0 There is insufficient evidence to suggest that the Poisson model is not a good fit for the **number of coins** found	**B1** **B1FT** **M1** **A1** **M1** **A1** **[6]**	**1.2** **1.1** **3.4** **1.1** **1.1** **2.2b**	Accept similar wording e.g.: **FT** (1.354 + their D3) 4 degrees of freedom *their TS* compared correctly to *their CV** and conclusion For non-assertive conclusion in **context**	'a suitable model' 'an appropriate model' **their CV* must be from upper tail of χ^2 table Allow "Accept H_0" For **A1** TS & CV must be correct

Question			Answer	Marks	AOs	Guidance	
	(e)		$\lambda = 1.98 + 0.42$ (= 2.4) used soi P(3) = 0.2090 awrt	**M1** **A1** **[2]**	**3.1b** **1.1**	(0.209014…)	N.B. 0.209 only is **A0**
	(e) alt		*p(C=0)×p(J=3) + p(C=1)×p(J=2) + p(C=2)×p(J=1) + p(C=3)×p(J=0)* *0.138069×0.008113 + 0.273377×0.057952 + 0.270643×0.275960 + 0.178625×0.657047* *0.2090*	**M1** **A1** **[2]**	**3.1b**	*0.00112 + 0.01584 + 0.07469 + 0.11737*	*N.B. 0.209 only is **A0***
	(f)		Po(10 × their λ from **(e)**) used soi (their $\lambda \neq 1.98$) P(≥ 30) = 1 – 0.8679 = 0.1321 awrt	**M1** **A1** **[2]**	**3.3** **3.4**	Following **M0** allow **SC1** for Normal approximation leading to answer rounding to 0.13	

Question			Answer	Marks	AOs	Guidance	
5	(a)		Scatter diagram appears to be roughly **elliptical** the distribution may be **bivariate Normal**.	**E1** **E1** **[2]**	**3.5a** **2.4**	For either elliptical or bi-Norm	Normal bivariate is **E0** "**the data** is bivariate Normal" is **E0**
	(b)		$S_{sp} = 6009.1 - \frac{1}{15} \times 155.4 \times 518.9 = (633.296)$ oe $S_{ss} = 2322.7 - \frac{1}{15} \times 155.4^2 = (712.756)$ oe $S_{pp} = 21\,270.5 - \frac{1}{15} \times 518.9^2 = (3320.019...)$ oe $r = \frac{S_{sp}}{\sqrt{S_{ss}S_{pp}}} = \frac{633.3}{\sqrt{712.8 \times 3320.0}}$ $= 0.41$	**M1** **M1** **M1** **A1** **[4]**	**1.1a** **1.1** **1.1** **1.1**	For either S_{sp} For either S_{ss} or S_{pp} For general form including square root Allow full credit for correct answer even if no working	(0.4116859…) If given to more than 2sf must be rounded correctly
	(c)		H_0: $\rho = 0$ H_1: $\rho > 0$ (one-tailed test) where ρ is the (population) correlation coefficient **between *s* and *p*** (Critical value =) 0.4409 0.41 < 0.4409 Insufficient evidence to reject H_0 There is insufficient evidence at the 5% level to suggest that there is **positive** correlation between sulphur dioxide and PM10 levels.	**B1** **B1** **B1** **M1** **A1** **[5]**	**3.3** **2.5** **3.4** **1.1** **2.2b**	For both hypotheses For defining ρ For critical value *their TS* compared correctly to *their CV* and conclusion For non-assertive conclusion in **context**	H_0: no correlation in the **population** H_1: positive correlation in the **population** ±0.4409 is **B0** "Accept H_0" is **M0** For **A1** TS & CV must be correct

Question			Answer	Marks	AOs	Guidance	
	(d)		A random sample enables proper **inference** **about the population** to be undertaken A random **sample** is **unbiased** For the **hypothesis test** to be **valid** it is necessary to assume that the sample is random	**B1** **B1** **[2]**	**2.4** **2.4**	Any two of the statements in bold (**B1** each)	"a random sample **reduces** bias" is **B0** "data is not biased" is **B0**
	(e)		Prediction for n = 150 is 38 awrt	**B1** **[1]**	**1.1**	(37.96)	
	(f)		Although it is **interpolation**, r^2 is only 0.3056 oe the points do not lie very close to the line **Not (very) reliable**	**E1** **E1** **[2]**	**3.5a** **3.5a**	For either but must conclude about reliability Mark the last statement about reliability	Weak correlation *r = 0.5528*

Question			Answer	Marks	AOs	Guidance	
6	(a)		$P(X < \frac{3}{2}n) = \frac{1}{2}$	B1 [1]	1.1		
	(b)		X can take $n+1$ values soi of which $\frac{1}{2}n$ are below $\frac{3}{2}n$ $P(X < \frac{3}{2}n) = \frac{n}{2(n+1)} = \frac{n}{2n+2}$	M1 M1 A1 [3]	3.1a 1.1 1.1	 convincing reasoning required if using algebraic approach e.g. $(\frac{3}{2}n-1)-(n-1)$ $\frac{1}{2}(n+1-1)$ $\frac{1}{2}(n+1)-\frac{1}{2}$	$2n-(n-1)$ accept correct conclusion reached from list(s) of numbers **SC3** for correct expression with no working
	(c)		$(n+1)$ values so $\text{Var}(X) = \frac{1}{12}[(n+1)^2 - 1]$ $\text{Var}(Y) = 6 \times \textit{their}\ \text{Var}(X)$ $\text{Var}(Y) = \frac{1}{2}(n^2+2n) = \frac{1}{2}n(n+2)$ isw	M1 M1 A1 [3]	2.2a 1.1 1.1	Use of discrete uniform variance over $n+1$ values Simplified form	$\frac{1}{12}(n^2+2n)$

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